Nothing special about general relativity?*

Arno Keppens (arnok@oma.be)

Space Pole, Circular Avenue, 1180 Brussels, Belgium

March 27, 2025

Abstract

It is argued that the general theory of relativity does not apply to spacetime only, but to all effective fields whose constituent velocities vary negatively with densities, putting the theory within a class of more broadly applicable theoretical frameworks like thermodynamics and fluid dynamics. The 2+1-dimensional dynamics of festival attendees going to a bar for drinks serves as a demonstrator.

*Essay written for the Gravity Research Foundation 2025 Awards for Essays on Gravitation.

General relativity

Albert Einstein presented "the field equations of gravitation" to the Prussian Academy of Sciences in three stages in 1915 (Sitzungsber. p. 778, p. 799, and p. 844). Less than four months after the last stage (November 25, 1915), his overview article on "the foundation of the general theory of relativity" appeared in Annalen der Physik (March 20, 1916), within one week after his 37th anniversary. One could argue this to be the most beautiful birthday gift ever, both to and from Einstein, within the physics community and far beyond. In it, he phrased the general principle of relativity, stating that *the laws of physics must be of such a nature that they apply to systems of reference in any kind of motion* (translation of first occurrence). As Einstein demonstrated in the same article, this principle directly imposes the theory of relativity upon each observer, essentially as a theory of spacetime coordinate transformations that do not affect reference systems' (observed) dynamics (requirement of general covariance). In order get there, however, it's clear that one needs a coordinate transformation gauge (anchoring point) that comes as a replacement of absolute Newtonian spacetime. Einstein borrowed this gauge from his special theory of relativity (1905), which considered the speed of light appearing in Maxwell's equations as an observer-independent quantity, and which was henceforth assumed to generally hold in the limit of infinitesimal spatiotemporal scales. One thus has for two reference systems S and S' that

$$\frac{dr}{dt} = c = \frac{dr'}{dt'} \tag{1}$$

indicating that both systems observe the same light speed c, measured by the distance that light has traveled along the spatial line element dr or dr' within a time interval dt or dt', respectively, irrespective of the two systems' relative movement. This entails that one can define a system-independent spatiotemporal line element ds with invariable length:

$$(cdt)^2 - (dr)^2 = ds^2 = (cdt')^2 - (dr')^2 \tag{2}$$

or, in generalized coordinate notation x_μ and for arbitrary orientations of the three-dimensional spatial reference frames:

$$ds^2 = \sum_\mu (dx_\mu)^2 = \sum_\mu (dx'_\mu)^2 \tag{3}$$

with the index μ ranging over all four spacetime dimensions. Einstein continued by posing that any coordinate transformation can be represented by definite linear homogeneous expressions (here jointly represented by a 4×4 tensor α) between the one-dimensional line elements, so for a transformation from S to S' one has:

$$dx'_\mu = \sum_\nu \alpha_{\mu\nu} dx_\nu \tag{4}$$

Insertion of Eq. (4) into the right hand side of Eq. (3) yields for the invariant spatiotemporal line element:

$$ds^2 = \sum_{\mu\nu} g_{\mu\nu} dx_\mu dx_\nu \tag{5}$$

with the now symmetrical tensor $g_{\mu\nu}$ containing functions of x_μ that are independent of the reference frame S'. It was exemplary of Einstein's genius to realize that if S' is chosen to be a local force-free (geodesic) reference frame, then the gravitationally induced dynamics of S' with respect to a distant observer S is fully captured by the dynamics of the so-called metric tensor $g_{\mu\nu}$, while essentially nothing more happens than introducing an acceleration into a coordinate transformation. As the latter only involves a transformation of spatial and temporal line elements, it can be interpreted by the distant observer as a 'curvature' of spacetime itself wherein S' moves linearly. Equations of motion are, although not straightforwardly, obtained from exploiting the invariance of the spatiotemporal line element, $\delta \int ds = 0$, and conservation laws.

Concerns

Einstein's two theoretical leaps in the development of the general theory of relativity, i.e., (1) having the speed of light as a gauge for generally covariant coordinate transformations and (2) identification of the dynamics of the metric tensor with gravitational spacetime curvature, can however be questioned. For example, can one select a different reference (velocity) gauge for the coordinate transformations? Or, regarding the second point, can (accelerations induced by) other forces be captured by metric tensors as well? For several historical reasons, including the theoretical and experimental successes of electromagnetism, the first question has been of little interest to physicists, while the second was addressed by theorists already soon after Einstein's seminal work, also by himself (1919). Probably the most well-known instance thereof is the extension of the metric tensor to five spatiotemporal dimensions by Theodor Kaluza (1921) and Oskar Klein (1926), allowing for the integration of Maxwell's electrodynamics into the full field equations. This was a beautiful achievement in itself, which even initiated many of the later quantum field theory developments, especially from Klein's suggestion that the additional spatial dimension is curled-up. Recently, although obviously based on lots of intermediate theory building, Langa Horoto and Frederik Scholtz (2024) have motivated how the theory of Kaluza and Klein can be extended to the entire Standard Model, including the Brout–Englert–Higgs mechanism for mass acquisition. This, in its turn, raises the question whether the general theory of relativity might be applicable to other, non-fundamental interactions as well. In the next sections, I will argue this to be the case indeed, if one is willing to reconsider (1) at the same time, i.e., if one allows other interaction velocities to act as coordinate transformation gauges.

Gravitating systems

The most striking feature of the general theory of relativity is that it only adheres to the dynamics of spatiotemporal line elements – or, behind that, spacetime coordinate transformations – to account for gravitational effects. This direct connection is actually enabled by the equivalence principle, which does not allow distinguishing between a constant acceleration and a gravitational field. If, however, in line with present-day quantum gravity approaches, spacetime indeed consists of pre-geometric Planck-scale degrees of freedom (here called constituents), then a flat spacetime background will locally appear as a spacetime constituent density gradient to an accelerated test particle. Therefore, as especially Thanu Padmanabhan (throughout his life and until his last publication, 2022) and myself (2018, 2023) have argued, the equivalence principle dictates that the gravitational field is an effective spacetime constituent density gradient field, cast atop a (roughly) constant constituent density background. This effective field perspective on spacetime immediately explains the success of gravity studies in terms of fluid dynamics and thermodynamics, again see the extensive work of Padmanabhan and references therein. Moreover, it provides a straightforward interpretation of the spacetime constituent *interaction* that *induces* spacetime curvature. In order to demonstrate this, consider a spherically symmetric scenario with a central gravitational field source and a constant spacetime background. Eq. (5) hence reduces to a Schwarzschild-de Sitter-like metric:

$$ds^2 = -f(r)(cdt)^2 + f(r)^{-1}dr^2 + d\Omega^2 \tag{6}$$

with c the speed of light as the observer-invariant spacetime constituent velocity. This line element definition holds for any number of spatial dimensions larger than one if all angular dimensions are captured by Ω. One then has a general solution for the function $f(r)$ that captures the spacetime curvature with radial distance:

$$f(r) = 1 - \frac{\alpha}{r} - \beta(r) \tag{7}$$

Here, α represents a reference radius that is determined by the strength of the central source and β adds the background. Rewriting Eq. (7) in terms of radial spacetime constituent densities $\rho_r \propto r^{-1}$, one obtains:

$$f(\rho_r) = 1 - \frac{\rho_r}{\rho_\alpha} - \frac{\rho_\beta}{\rho_\alpha} \tag{8}$$

with ρ_α a reference density corresponding to $r \equiv \alpha$ and ρ_β the constant radial background density, appropriately scaled, making the last term now independent of r. At each point in space with $\alpha \leq r \leq \infty$, the local constituent density is thus given by the sum of a source term $\rho_r(r)$ and a constant radial background term ρ_β. Fully in line with Einstein's discovery (1911, again from the equivalence principle, yet even before his development of the general theory of relativity) that the speed of light depends on the gravitational field strength to a non-geodesic observer, we thus must here conclude that the local constituent velocity depends on the (total) local constituent density to a remote observer that resides at a fixed distance from the central source. For radially moving constituents ($ds^2 = 0$ and $d\Omega^2 = 0$), the effective constituent velocity $c = dr/dt$ as determined from Eqs. (6) and (8) by the non-geodesic observer indeed simplifies to:

$$c(\rho_r) = c_0 \left(1 - \frac{\rho_r + \rho_\beta}{\rho_\alpha}\right) \tag{9}$$

with c_0 the maximum possible constituent speed in the absence of others (cf. movement in Fock vacuum). Therefore, while the reference speed c is invariant to each observer locally, the constituent speed slows down towards the source, i.e., for increasing constituent densities, to the stationary observer. We will hence call "gravitating systems" all those effective fields (or fluids) whose local constituent velocities vary negatively with local constituent densities: $\delta c \propto -\delta\rho$. The theory of general relativity then applies to all these systems, upon considering the constituent velocity as an observer-invariant reference speed that acts as a gauge for the measurement and transformation of spatial and temporal distances.

Bar dynamics

Consider yourself in an observation tower located near a large festival ground. Towards the middle of the ground, the organizers have installed a large circular bar, where all festival attendees must go for drinks. You, however, are a non-thirsty (for now) observer that resides in the tower and hence does not participate in the festival constituent dynamics that you contemplate. Then, upon the presence of a sufficient number of attendees, you would start to see a rather continuous and roughly radial in- and outflow of them to and from the bar. Yet at the bar itself ($r = \alpha$), the radial velocity of the constituents is zero (at least for the time to get a drink). So a festival attendee approaching the bar from a large distance will, as seen by you from the tower, make a way through the increasing people density ever more slowly. In other words, on average (as is also the case for gravity, given quantum fluctuations of spacetime) the festival constituent speed decreases with increasing density and one can expect a variation on Eq. (9) to apply. A random attendee, however, upon being asked about bar queuing experiences, might indicate to have just moved in and out with the crowd, while roughly keeping the same pace as close-by attendees at every moment. Therefore, a topological description of the bar dynamics could make reference to the apparently locally invariant constituent speed for constructing equations of motion. It becomes clear that the latter approach will not differ substantially from the general theory of relativity, apart from the identification of the effective field constituents and, for this Schwarzschild-de Sitter-like scenario, the central attractor. Grasping how a test mass, understood as a constituent condensate with a timelike line element, say two people carrying in a new barrel of beer in this example, is accelerated towards the attractor is more difficult (although demonstrated by simulations for gravity in own work, 2023). On the other hand, it is clear that from a certain distance from the bar ($r = 3\alpha$ for the Schwarzschild metric) the condensate will have to slow down and cannot but move with the crowd...

Conclusions

We have identified gravitating systems as those effective fields whose local constituent velocities vary negatively with local constituent densities (to stationary observers) and have argued that the general theory of relativity is not only a background-independent, but also a constituent-independent theory that applies *mutatis mutandis* to all of these systems, when using the local constituent movement as a gauge for (transformations between) distance and time measurements by different observers. This rightfully puts the general theory of relativity within a class of broadly applicable theoretical frameworks like thermodynamics and fluid dynamics. Actually, gravity may well be the simplest instantiation of a gravitating system, with $\delta c = -\delta\rho$ up to a constant from Eq. (9), while one can imagine more exotic relations applying to other systems. This has its implications for the development of theories of quantum gravity: spacetime merely consists of an effective field of interacting constituents (too), which can be assumed to be of the order of the Planck scale, and hence must have a valid thermodynamic description just as well (again see recent works by Padmanabhan, 2022, and myself, 2024, in honor of 50 years of black hole thermodynamics). On the other hand, again with variations on Kaluza-Klein theory in mind, the unification with other fundamental forces requires additional field properties, possibly captured in the dynamics of curled-up dimensions. Think of two people performing a spinning dance by holding hands while heading towards the bar, thus affecting festival attendees in their vicinity. Anyhow, next time you move with the crowd when going for a drink, imagine that you reside on a geodesic trajectory just as well.

References

Einstein, Albert (1905): Zur Elektrodynamik bewegter Körper. Ann. Phys. 17.

Einstein, Albert (1911): Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes. Ann. Phys. 35.

Einstein, Albert (1916): Die Grundlage der allgemeinen Relativitätstheorie. Ann. Phys. 49.

Einstein, Albert (1919): Spielen die Gravitationsfelder im Aufbau der materiellen Elementarteilchen eine wesentliche Rolle? Sitzungsber. Preuss. Akad. Wiss. Berlin: 140–146.

Kaluza, Theodor (1921): Zum Unitätsproblem in der Physik. Sitzungsber. Preuss. Akad. Wiss. Berlin: 966–972.

Klein, Oskar (1926): Quantentheorie und fünfdimensionale Relativitätstheorie. Z. Phys. A 37(12): 895–906. doi:10.1007/BF01397481.

Keppens, Arno (2018): What Constitutes Emergent Quantum Reality? A Complex System Exploration from Entropic Gravity and the Universal Constants. Entropy 20(5): 335. doi:10.3390/e20050335.

Padmanabhan, Thanu and Chakraborty, Sumanta (2022): Microscopic origin of Einstein's field equations and the raison d'être for a positive cosmological constant. Phys. Lett. B 824: 136828. doi:10.1016/j.physletb.2021.136828.

Keppens, Arno (2023): Emergent Gravity Simulations for Schwarzschild–de Sitter Scenarios. Foundations 3(2): 231-240. doi:10.3390/foundations3020019.

Horoto, Langa and Scholtz, Frederik G. (2024): A new perspective on Kaluza–Klein theories. Ann. Phys. 469: 169748. doi:10.1016/j.aop.2024.169748.

Keppens, Arno (2024): Schwarzschild spacetime thermodynamics. arXiv:submit/5350485.